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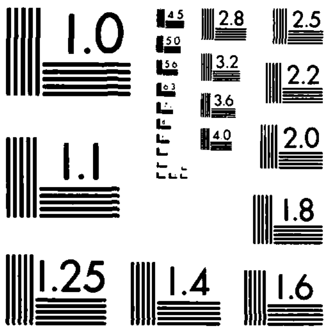

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RESISTANCE CURVES FOR CRACK GROWTH UNDER PLANE-STRESS CONDITIONS IN AN ELASTIC PERFECTLY-PLASTIC MATERIAL

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Abstract

A recently developed solution for ~~the~~ a plastic strain, $\epsilon_y^P(x,t)$, on the crack line is used in conjunction with a critical strain criterion to construct curves for $K_R(a)$ versus a, where a is the increase in crack length. Resistance curves have been computed for various values of the critical plastic strain. They show a monotonic increase of $K_R(a)$ with increase in crack length, to a constant steady-state value.

Additional keywords: Crack propagation; plane stress; Strain field.

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1. Introduction

For small-scale yielding, the condition for continued crack advance may be written as

$$K = K_R(a) \;, \tag{1.1}$$

where K on the left-hand side is regarded as the "applied" K, and a in $K_R(a)$ is the <u>increase</u> in crack length. The curve of $K_R(a)$ versus a is called the resistance curve or R-curve. Under plane stress conditions, $K_R(a)$ tends to be a monotonically increasing function of a, at least for many metals. Following the notation of Ref.[1,p.20], we will use K_{IC} to denote the value of K for initiation of crack propagation, i.e., $K_{IC} = K_R(0)$.

In this paper we consider a general geometry such as an edge crack in a thin sheet, and we construct resistance curves. These curves are based on a critical strain criterion for crack propagation, which stipulates that crack growth will proceed when the plastic strain on the crack line maintains a critical strain level ε_y^{Pf} at a distance x_f ahead of the crack tip. The critical strain criterion was proposed by McClintock and Irwin [2] and it has been used frequently for the Mode-III case. Rice [3,p.281] presented an expression for the plastic shear strain on the crack line for Mode-III crack propagation in an elastic perfectly-plastic material. It was also shown in Ref.[3] that the fracture criterion of critical plastic shear strain leads to an integral equation for the plastic zone size required for quasi-static crack extension.

The construction of curves for $K_R(a)/K_{IC}$ presented in this paper for plane stress, follows in general outline the method of Ref.[3]. The construction is based on an expression for the crack-line strain which was recently presented by Achenbach and Dunayevsky [4] and Achenbach and Li [5]. As shown in some detail in Ref.[5], in the plastic loading zone the stresses and strains can be expanded in powers of the distance, y, to the crack line. Substitution of the expansions into the equilibrium equations, the yield condition and the constitutive equations, yields a system of simple ordinary differential equations for the coefficients of the expansions. This system is solvable if it is assumed that the stress σ_y is uniform on the crack line. By matching the relevant stress components and particle velocities to the dominant terms of appropriate elastic fields at the elastic-plastic boundary, a complete solution was obtained for the plastic strain, ε_y^P, in the plane of the crack. The solution depends on position on the crack-line and time, and applies from the propagating crack tip up to the moving elastic-plastic boundary.

It is shown in this paper that the critical strain criterion yields an integral equation for $x_p(a)$, where $x_p(a)$ is the extent of the plastic zone on the crack line as a function of the increase in crack length. The integral equation has been solved numerically for various values of $\varepsilon_y^{Pf}/(\varepsilon_y)_{PB}$, where $(\varepsilon_y)_{PB}$ is the crack-line strain at the elastic-plastic boundary. A relation between K_I and $x_p(a)$ subsequently yields resistance curves $K_R(a)$.

The geometry is shown in Fig. 1. The x_3-axis of a stationary coordinate system is parallel to the crack front, and x_1 points in the direction of crack growth. The position of the crack tip is defined by $x_1 = a(t)$. A moving coordinate system, x,y,z, is centered at the crack tip, with its axes parallel to the x_1,x_2 and x_3 axes.

2. Crack-Line Strain

For monotonic loading, expressions for the strain rate $\dot{\varepsilon}_y(x_1,t)$, on the crack line ahead of a propagating crack tip, are given in Refs.[4] and [5] as

$$\frac{E}{k}\dot{\varepsilon}_y(x_1,t) = \frac{2\dot{a}(t)}{x_1-a(t)}\,\ell n\,\frac{x_p(t)}{x_1-a(t)} + \frac{B_1\dot{a}(t)+B_2\dot{x}_p(t)}{x_1-a(t)} + \frac{C_1\dot{a}(t)+C_2\dot{x}_p(t)}{x_p^3(t)}[x_1-a(t)]^2, \quad (2.1)$$

where x_1 defines the point of observation in the stationary coordinate system, while $a(t)$ defines the position of the crack tip and $x_p(t)$ is the size of the plastic zone along the crack line. Also, k is the yield stress in shear, E is Young's modulus, and the constants B_1, B_2, C_1 and C_2 have been derived in Ref.[4] as

$$B_1 = \frac{1}{8}(1+\nu)[(\kappa + 5) + 2(\kappa + 1)\sqrt{2}] - \frac{2}{3} \quad (2.2)$$

$$B_2 = \frac{1}{8}(1+\nu)[(\kappa + 5) + (\kappa + 1)\sqrt{2}] \quad (2.3)$$

$$C_1 = \frac{1}{8}(1+\nu)[-(\kappa + 5) + (\kappa + 1)\sqrt{2}] + \frac{2}{3} \quad (2.4)$$

$$C_2 = \frac{1}{8}(1+\nu)[-(\kappa + 5) + 2(\kappa + 1)\sqrt{2}] , \quad (2.5)$$

where ν is Poisson's ratio and κ is defined as $(3-\nu)/(1+\nu)$.

In the stationary coordinate system, (2.1) can be integrated to yield

$$\varepsilon_y(x_1,t) = (\varepsilon_y)_{PB} + \varepsilon_y^P(x_1,t) , \quad (2.6)$$

where $(\varepsilon_y)_{PB}$ is the elastic strain at the elastic - plastic boundary:

$$(\varepsilon_y)_{PB} = \frac{k}{E}(2-\nu) , \quad (2.7)$$

$\varepsilon_y^P(x_1,t)$ is the plastic strain

$$\varepsilon_y^P(x_1,t) = \int_{t_p}^{t} \dot{\varepsilon}_y(x_1,s)ds \tag{2.8}$$

The lower limit t_p, which is the time that the elastic-plastic boundary reaches position x_1 and the plastic strain starts to accumulate, follows from

$$a(t_p) + x_p(t_p) = x_1 \tag{2.9}$$

By integration by parts of the terms multiplying B_2 and C_2, $\varepsilon_y^p(x_1,t)$ can be separated into two components:

$$\varepsilon_y^p(x_1,t) = \varepsilon_y^{sP}[\xi(x_1,t)] + \int_{t^*}^{t} \dot{\varepsilon}_y^{pP}(x_1,s)ds, \tag{2.10}$$

where

$$\frac{E}{k}\,\varepsilon_y^{sP}[\xi(x_1,t)] = B_2(\xi-1) - \frac{1}{2}\,C_2\,(\frac{1}{\xi^2} - 1) \tag{2.11}$$

$$\xi(x_1,t) = x_p(t)/[x_1-a(t)]\ , \tag{2.12}$$

and

$$\frac{E}{k}\,\dot{\varepsilon}_y^{pP}(x_1,s) = \dot{a}(s)\ f[x_1-a(s),x_p(s)] \tag{2.13}$$

$$f[x_1-a(s),x_p(s)] = \frac{1}{x_1-a(s)}\ \{2\ln\xi(x_1,s) + B_1-B_2\xi(x_1,s) + \frac{C_1}{[\xi(x_1,s)]^3} - \frac{C_2}{[\xi(x_1,s)]^2}\} \tag{2.14}$$

Here we have used (2.9) at the lower limits of integration of Eq.(2.8). The lower limit t^* in (2.10) is defined as $t^* = t_f$ for $t_f > t_p$, where t_f is the time that crack propagation starts. When $t_p > t_f$, the lower limit is defined as $t^* = t_p$.

We will now assume that crack propagation is governed by the criterion of a critical plastic strain at a fixed micro-structural distance x_f ahead of the crack tip, i.e., at $x_1 = a(t) + x_f$. The condition for stable crack propagation then is

$$\varepsilon_y^{Pf} = \varepsilon^{sP}[x_p(t)/x_f] + \int_{t^*}^{t} \dot{\varepsilon}_y^{pP}[a(t) + x_f, s]\, ds, \tag{2.15}$$

where ε_y^{Pf} is the critical value of the plastic strain, and the right-hand side follows from (2.10). Equation (2.15) is an equation for $x_p(t)$. For the case $t_f < t_p$, the integral in (2.15) vanishes when the upper limit is taken as $t = t_f$, and the following equation for $x_p(t_f)$ is obtained

$$\frac{B_2}{x_f} x_p^3(t_f) + [\frac{1}{2} C_2 - B_2 - \frac{E}{k} \varepsilon_y^{Pf}]\, x_p^2(t_f) - \frac{1}{2} C_2 x_f^2 = 0 \tag{2.16}$$

A convenient form of (2.15) can be obtained by replacing the independent variable t by crack length a. Thus we consider the strain as well as the position of the elastic-plastic boundary as functions of a. After introducing the normalizations

$$\bar{a} = a/x_f \ , \quad \bar{x}_p(\bar{a}) = x_p[t(a)]/x_f \ , \tag{2.17a,b}$$

we can write

$$\varepsilon_y^{Pf} = \varepsilon^{sP}[\bar{x}_p(\bar{a})] + \int_{a^*}^{\bar{a}} f[\bar{a} + 1 - \eta, x_p(\eta)]\,d\eta \tag{2.18}$$

where

$$f[\bar{a} + 1 - \eta, x_p(\eta)] = \frac{1}{\bar{a}+1-\eta}\,[2\ell n\zeta + B_1 - B_2\zeta + \frac{C_1}{\zeta^3} - \frac{C_2}{\zeta^2}] \tag{2.19}$$

$$\zeta = x_p(\eta)/(\bar{a} + 1 - \eta) \tag{2.20}$$

The lower limit a^* in Eq.(2.18) is defined by

$$a^* = 0 \ , \qquad \text{when} \qquad \bar{a} + 1 \leq \bar{x}_p(0) \tag{2.21}$$

and

$$a^* + \bar{x}_p(a^*) = \bar{a} + 1, \quad \text{when} \quad \bar{x}_p(0) \leq \bar{a} + 1 \ , \tag{2.22}$$

where $\bar{x}_p(0) = x_p(t_f)x_f$. Equation (2.21) corresponds to $t_f > t_p$, while (2.22) corresponds to $t_f < t_p$.

Equation (2.18) is a Volterra integral equation for $\bar{x}_p(\bar{a})$, which can be solved by a step-by-step procedure. If $\bar{x}_p(\bar{a})$, is known for $0 \leq \bar{a} \leq \bar{a}_1 < \bar{x}_p(0) - 1$, then for $\bar{a} = \bar{a}_1 + \Delta < \bar{x}_p(0) - 1$, Eq.(2.18) yields

$$\varepsilon_y^{sP}[\bar{x}_p(\bar{a}_1+\Delta)] = \varepsilon_y^{Pf} - \int_o^{\bar{a}_1+\Delta} f[\bar{a}_1+\Delta+1-\eta, \bar{x}_p(\eta)]\,d\eta \tag{2.23}$$

where Δ is small. The integral in (2.21) can be approximated by

$$\int_o^{\bar{a}_1+\Delta} f[\bar{a}_1+\Delta+1-\eta, \bar{x}_p(\eta)]\,d\eta \cong \int_o^{\bar{a}_1} f[\bar{a}_1+\Delta+1-\eta, \bar{x}_p(\eta)]\,d\eta + f(\bar{a}_1+\Delta+1-\eta, \bar{x}_p(\bar{a})]\Delta \tag{2.24}$$

The integral on the right-hand-side of (2.24) is known. Substitution of (2.24) in (2.23) yields a cubic equation for $\bar{x}_p(\bar{a}_1+\Delta)$, which can be solved. This value for $\bar{x}_p(\bar{a}_1+\Delta)$ is used as the starting point for an iteration procedure whereby subsequent values of $\bar{x}_p(\bar{a}_1+\Delta)$ are substituted in the integral in

Eq.(2.21) to yield improved values of $\bar{x}_p(\bar{a}_1+\Delta)$, until a desired accuracy has been achieved. For the first step in this procedure we use $\bar{x}_p(\Delta) \cong \bar{x}_p(0) + \bar{x}'_p(0)\Delta$, where $\bar{x}_p(0)$ follows from (2.16), and $\bar{x}'_p(0) = d\bar{x}_p(\bar{a})/d\bar{a}$ at $\bar{a} = 0$, can be obtained from (2.18).

For the case when $\bar{x}_p(0) - 1 \leq \bar{a}$, defined by (2.22), the lower limit of the integral is a function of the upper limit, $\bar{a}$, and of the unknown function $\bar{x}_p(\bar{a})$. Again, if $\bar{x}_p(\bar{a})$ is known for $\bar{a} \leq \bar{a}_1$, then for $\bar{a} = \bar{a}_1 + \Delta$, Eq. (2.22) yields $a^* + \bar{x}_p(a^*) = \bar{a}_1 + \Delta + 1$, where $\bar{x}_p(a^*)$ is known since $a^* < \bar{a}$. Hence we can solve $a^* = a^*(\bar{a}_1,\Delta)$. Using an analogous method as before, Eq.(2.18) can subsequently be solved for $\bar{x}_p(\bar{a}+\Delta)$.

In the actual computations the following values of the relevant parameters have been considered:

$$\varepsilon_y^f \equiv \varepsilon_y^{Pf}/(\varepsilon_y)_{PB} = 2,\ 6, \text{ and } 10 \tag{2.25}$$

The Poisson's ratio was taken as $\nu = 0.3$. For $\bar{a} = 0$, the value of $\bar{x}_p(0)$ follows directly by computation of the real root of Eq.(2.16), since $\bar{x}_p(0) = x_p(t_f)/x_f$. The numerical results show a subsequent increase of $\bar{x}_p(\bar{a})$ with $\bar{a}$, where $\bar{x}_p(\bar{a})$ approaches an asymptotic value for large $\bar{a}$.

For quasi-static steady-state crack extension the following relation was derived by Achenbach and Li [5]:

$$\varepsilon_y^P(x) = \frac{k}{E}\left\{[\ln(\frac{x}{x_p})]^2 - B_1\ln(\frac{x}{x_p}) - \frac{1}{3}\left[(\frac{x}{x_p})^3 - 1\right]\right\} \tag{2.26}$$

The critical strain criterion then yields the relation

$$\frac{E}{K}\varepsilon_y^{Pf} - [\ln(\bar{x}_p)]^2 - B_1\ln\bar{x}_p + \frac{1}{3}\left[(1/\bar{x}_p)^3 - 1\right] = 0 \tag{2.27}$$

Equation (2.27) can be solved for $\bar{x}_p$ to yield a result which is independent of crack-tip speed and loading state. This result is just the asymptotic value of $\bar{x}_p(\bar{a})$ as $\bar{a}$ increases.

A convenient form of the resistance curve shows the ratio K_I/K_{IC} versus a dimensionless crack length. Here K_I is the present stress intensity factor, and K_{IC} is the value of the stress intensity factor which is required to satisfy the fracture criterion for a stationary crack. In Ref.[5, Eq.57] an expression was presented which relates $x_p(t)$ to K_I/k. In the present notation this expression states

$$\bar{x}_p(\bar{a}) = \frac{2\sqrt{2}}{9}\,\frac{1}{\pi}\,\frac{1}{x_f}\,[K_I(\bar{a})/k]^2 \tag{2.28}$$

It follows that

$$K_I(\bar{a})/K_{IC} = K_R(\bar{a})/K_{IC} = [\bar{x}_p(\bar{a})/\bar{x}_p(0)]^{\frac{1}{2}} \tag{2.29}$$

The quantity x_f which enters in $\bar{a}$, can be eliminated by equating the result for $\bar{x}_p(0)$ obtained from (2.16) to $\bar{x}_p(0)$ as obtained from (2.28). By using the resulting expression for x_f in the definition of $\bar{a}$, as given by (2.17a), we find

$$\bar{a} = \frac{a}{x_f} = \frac{9\pi}{2\sqrt{2}}\,\frac{a\,\bar{x}_p(0)}{[K_{IC}/k]^2} \tag{2.30}$$

For the three values of ε_y^f given by Eq.(2.25), curves for $K_R(\bar{a})/K_{IC}$ versus $\bar{a}$ are shown in Fig. 2. It is noted that the resistance curves show a monotonic increase with increase in crack length, to a stable phase of crack propagation where $K_R(\bar{a})/K_{IC}$ assumes the steady-state value.

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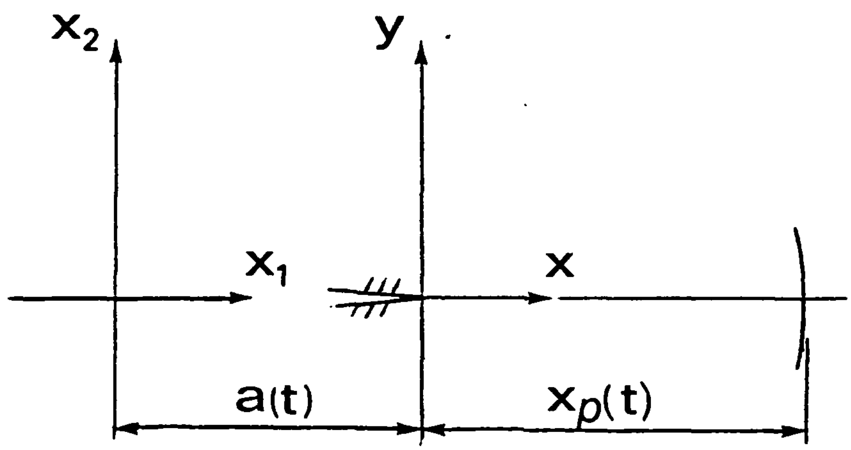

Fig. 1. Geometry of propagating crack; a(t) is increase in crack length; $x = x_p(t)$ defines the elastic-plastic boundary.

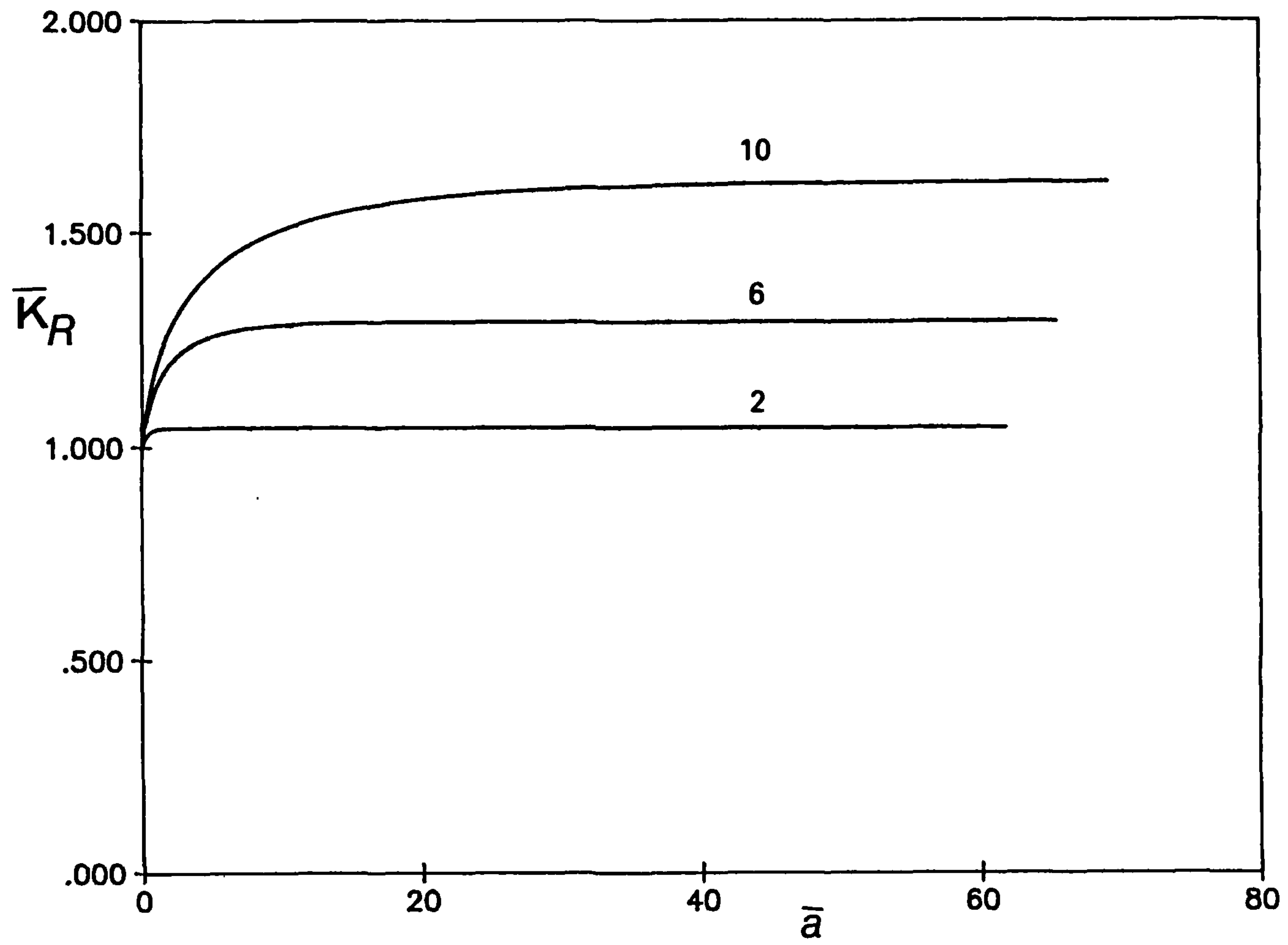

Fig. 2. Resistance curves for values of $\varepsilon_y^f \equiv \varepsilon_y^{Pf}/(\varepsilon_y)_{PB} = 2, 6$ and 10; $\overline{K}_R(\bar{a}) = K_R(\bar{a})\,/K_{IC}$; $\bar{a} = a(t)/x_f$.

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